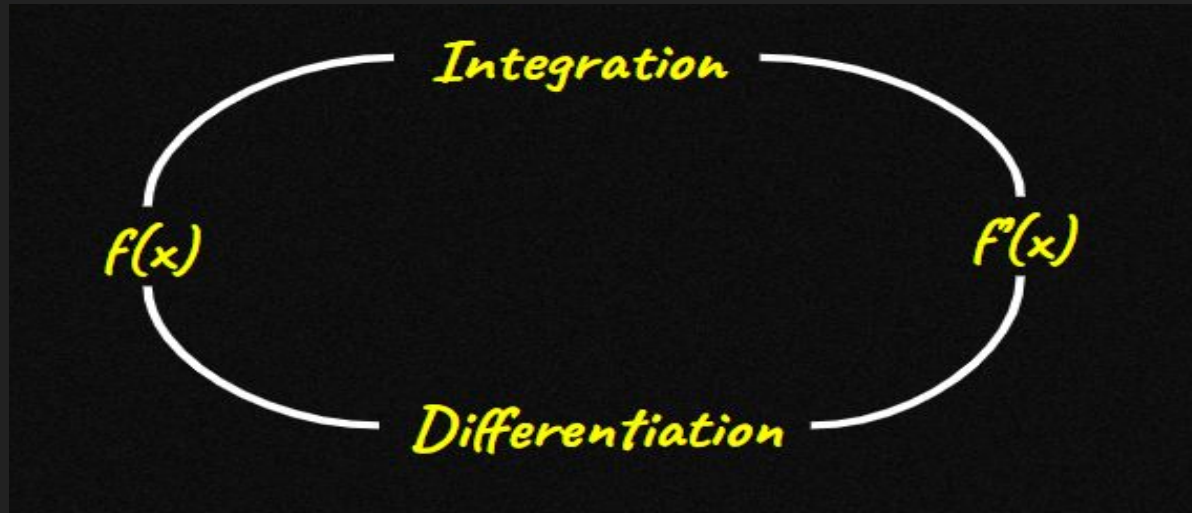


INTEGRATION

- Integration is the reverse process of differentiation.



INTEGRALS

INDEFINITE

DEFINITE

BASIC FORMULAS

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$$

$$\int dx = x + C$$

$$\int e^x dx = e^x + c,$$

$$\int e^{ax} dx = \frac{e^{ax}}{a} + c$$

USE MY CODE : SS12

BASIC FORMULAS

$$\int \frac{dx}{x} = \log x + c$$

$$\int a^x dx = a^x / \log_e a + c$$

USE MY CODE : SS12



BASIC PROPERTIES

$$\int c f(x) dx = c \int f(x) dx \text{ where } c \text{ is constant.}$$

$$\int \{ f(x) dx \pm g(x) \} dx = \int f(x) dx \pm \int g(x) dx$$

Example Find

$$(a) \int \sqrt{x} \, dx, \quad (b) \int \frac{1}{\sqrt{x}} \, dx,$$

$$(c) \int e^{-3x} \, dx \quad (d) \int 3^x \, dx \quad (e) \int x \sqrt{x} \, dx.$$

Example Evaluate the following integral:

i) $\int (x + 1/x)^2 dx$

ii) $\int \sqrt{x} (x^3+2x-3) dx :$

$$\text{iii)} \quad \int e^{3x} + e^{-4x} dx$$

$$\text{iv) } \int \frac{x^2}{x+1} dx =$$

v) $\int \frac{x^3 + 5x^2 - 3}{(x+2)} dx$

$$\int (2x + 3)^7 \, dx$$



SUBSTITUTION METHOD

$$\int \frac{x^3}{(x^2+1)^3} dx$$



FORMULAE

$$\text{a) } \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \frac{x-a}{x+a} + c$$

$$\text{b) } \int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \frac{a+x}{a-x} + c$$

FORMULAE

$$\text{c) } \int \frac{dx}{\sqrt{x^2 + a^2}} = \log \left| x + \sqrt{x^2 + a^2} \right| + c$$

$$\text{d) } \int \frac{dx}{\sqrt{x^2 - a^2}} = \log \left(x + \sqrt{x^2 - a^2} \right) + c$$

$$\text{e) } \int e^x \{f(x) + f'(x)\} dx = e^x f(x) + c$$



FORMULAE

$$\text{f) } \int \sqrt{x^2 + a^2} \, dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log(x + \sqrt{x^2 + a^2}) + c$$

$$\text{g) } \int \sqrt{x^2 - a^2} \, dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log(x + \sqrt{x^2 - a^2}) + c$$

Example

$$(a) \int \frac{e^x}{e^{2x} - 4} dx:$$

$$(b) \int \frac{1}{x + \sqrt{x^2 - 1}} dx:$$

$$(c) \int e^x (x^3 + 3x^2) dx:$$

INTEGRATION BY PARTS

$$\int u v dx = u \int v dx - \int \left[\frac{d(u)}{dx} \int v dx \right] dx$$

Evaluate

$$\text{i) } \int x e^x \, dx$$

$$\text{ii) } \int x \log x \, dx$$

$$\text{iii) } \int x^2 e^{ax} \, dx$$



DEFINITE INTEGRAL

$$\int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a).$$

USE MY CODE : SS12

Example

$$\int_0^2 x^5 \, dx$$

$$\int_1^2 (x^2 - 5x + 2)dx$$

PROPERTIES

$$(I) \quad \int_a^b f(x) dx = \int_a^b f(t) dt$$

$$(II) \quad \int_a^b f(x) dx = - \int_b^a f(x) dx$$

$$(III) \quad \int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx, \quad a < c < b$$

$$(IV) \quad \int_0^a f(x) dx = \int_0^a f(a-x) dx$$



PROPERTIES

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$\int_0^a f(x) dx = \int_0^a f(a-x) dx$$



PROPERTIES

$$\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$$

if $f(-x) = f(x)$

$$= 0$$

if $f(-x) = -f(x)$

Example

$$\int_0^2 \frac{x^2 dx}{x^2 + (2-x)^2}$$

CODE : SS12

Choose the most appropriate option (a) (b) (c) or (d).

Que. 1 Evaluate $= \int 5x^2 \, dx$:

(a) $5/3x^3 + k$

(b) $\frac{5x^3}{3} + k$

(c) $5x^3$

(d) None of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que. 2 Integration of $3 - 2x - x^4$ will become

(a) $-x^2 - x^5/5$

(b) $3x - x^2 - \frac{x^5}{5} + k$

(c) $3x - x^2 + \frac{x^5}{5} + k$

(d) None of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que. 3 Given $f(x) = 4x^3 + 3x^2 - 2x + 5$ and $\int f(x) dx$ is

(a) $x^4 + x^3 - x^2 + 5x$

(b) $x^4 + x^3 - x^2 + 5x + k$

(c) $12x^2 + 6x - 2x^2$

(d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que. 4 Evaluate $\int (x^2 - 1) dx$

(a) $x^5/5 - 2/3 x^3 + x + k$

(b) $\frac{x^3}{3} - x + k$

(c) $2x$

(d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que. 5 $\int (1 - 3x)(1 + x) dx$ is equal to

(a) $x - x^2 - x^3$

(b) $x^3 - x^2 + x$

(c) $x - x^2 - x^3 + k$

(d) None of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que. 6 $\int [\sqrt{x} - 1/\sqrt{x}] dx$ is equal to

(a) $\frac{2}{3}x^{3/2} - 2x^{1/2} + k$

(b) $\frac{2}{3}\sqrt{x} - 2\sqrt{x} + k$

(c) $\frac{1}{2\sqrt{x}} + \frac{1}{2x\sqrt{x}} + k$

(d) None of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que. 7 The integral of $px^3 + qx^2 + rk + w/x$ is equal to

- (a) $px^2 + qx + r + k$
- (b) $px^3/3 + qx^2/2 + rx$
- (c) $3px + 2q - w/x^2$
- (d) None of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que. 8 Use method of substitution to integrate the function $f(x) = (4x + 5)^6$ and the answer is

(a) $\frac{1}{28} (4x + 5)^7 + k$

(b) $(4x + 5)^7 / 7 + k$

(c) $(4x + 5)^7 / 7$

(d) None of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que. 9 Use method of substitution to evaluate $\int x(x^2 + 4)^5 dx$ and the answer is

- (a) $(x^2 + 4)^6 + k$
- (b) $1/12 (x^2 + 4)^6 + k$
- (c) $(x^2 + 4)^6 / + k$
- (d) None of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que. 10 Integrate $(x + a)^n$ and the result will be

(a) $\frac{(x + a)^{n+1}}{n + 1} + k$

(b) $\frac{(x + a)^{n + 1}}{n + 1}$

(c) $(x + a)^{n+1}$

(d) None of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que. 11 $\int 8x^2 / (x^3 + 2)^3 dx$ is equal to

(a) $-4/3(x^3 + 2)^2 + k$

(b) $-\frac{4}{3(x^3 + 2)^2} + k$

(c) $\frac{4}{3(x^3 + 2)^2} + k$

(d) None of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que. 12 Using method of partial fraction the integration of $f(x)$ when $f(x) = 1/x^2 - a^2$ and the answer is

(a) $\log x - \frac{a}{x+a} + k$

(b) $\log (x-a) - \log (x+a) + k$

(c) $\frac{1}{2a} \log \left(\frac{x-a}{x+a} \right) + k$

(d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que. 13 Use integration by parts to evaluate $\int x^2 e^{3x} dx$

(a) $x^2 e^{3x} / 3 - 2x e^{3x} / 9 + 2/27 e^{3x} + k$

(b) $x^2 e^{3x} - 2x e^{3x} + 2e^{3x} + k$

(c) $e^{3x} / 3 - x e^{3x} / 9 + 2e^{3x} + k$

(d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que. 14 $\int \log x \, dx$ is equal to

- (a) $x \log x + k$
- (b) $x \log x - x^2 + k$
- (c) $x \log x + k$
- (d) None of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que. 19 $\int x^x (1 + \log x) dx$ is equal to

(a) $x^x \log x + k$

(b) $ex^2 + k$

(c) $\frac{x^2}{2} + k$

(d) $x^x + c$



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Choose the most appropriate option (a) (b) (c) or (d).

Que. 20 If $f(x) = \sqrt{1+x^2}$ then $\int f(x)dx$ is

(a) $\frac{2}{3} x (1+x^2)^{3/2} + k$

(b) $\frac{x}{2} \sqrt{1+x^2} + \frac{1}{2} \log(x+\sqrt{x^2+1}) + k$

(c) $\frac{2}{3} x (1+x^2)^{3/2} + k$

(d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que. 21 $\int \frac{\sqrt{2}(x^2 + 1)}{\sqrt{x^2 + 2}} dx$ is equal to

(a) $\frac{x}{\sqrt{2}} \left(\sqrt{x^2 + 2} \right) + k$

(b) $\sqrt{x^2 + 2} + k$

(c) $1/(x^2 + 2)^{3/2} + k$

(d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que. 22 $\int (e^x + e^{-x})^2 (e^x - e^{-x}) dx$ is

(a) $\frac{1}{3}(e^x + e^{-x})^3 + k$

(b) $\frac{1}{2}(e^x - e^{-x})^2 + k$

(c) $e^x + k$

(d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que. 24 $\int x e^x / (x+1)^2 dx$ is equal to

(a) $e^x / (x+1) + k$

(b) $e^x / x + k$

(c) $e^x + k$

(d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que. 25 $\int (x^4 + 3/x) dx$ is equal to

(a) $x^5/5 + 3 \log |x|$

(b) $1/5 x^5 + 3 \log |x| + k$

(c) $1/5 x^5 + k$

(d) none of these



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CA FOUNDATION

CODE : SS12

Choose the most appropriate option (a) (b) (c) or (d).

Que. 26

Evaluate $\int_1^4 (2x + 5) dx$ and the value is

(a) 3

(b) 10

(c) 30

(d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que. 27 $\int_1^2 \frac{2x}{1+x^2} dx$ is equal to

- (a) $\log_e (5/2)$
- (b) $\log_e 5 - \log_e 2 + k$
- (c) $\log_e (2/5)$
- (d) None of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que. 28

$$\int_0^4 \sqrt{3x+4} \, dx \text{ is equal to}$$

- (a) $9/112$
- (b) $112/9$
- (c) $11/9$
- (d) None of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que. 29

$$\int_0^2 \frac{x+2}{x+1} dx \text{ is}$$

(a) $2 + \log_e 2$

(b) $2 + \log_e 3$

(c) $\log_e 3$

(d) None of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que. 30

The value of $\int_2^3 f(5-x) dx - \int_2^3 f(x) dx$ is

- (a) 1
- (b) 0
- (c) -1
- (d) None of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que. 31 $\int (x - 1)e^x/x^2 dx$ is equal to

(a) $e^x/x + k$

(b) $e^{-x}/x + k$

(c) $-e^x/x + k$

(d) None of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que. 32 $\int_0^2 3x^2 \, dx$ is

- (a) 7
- (b) -8
- (c) 8
- (d) None of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que. 33 Using integration by parts $\int x^3 \log x \, dx$

- (a) $x^4/16 + k$
- (b) $x^4/16 (4 \log x - 1) + k$
- (c) $4 \log x - 1 + k$
- (d) None of these



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CODE : SS12

Choose the most appropriate option (a) (b) (c) or (d).

Que. 34

Evaluate $\int \left(\frac{e^x - e^{-x}}{e^x + e^{-x}} \right) dx$ and the value is

(a) $\log_e |e^x + e^{-x}|$

(b) $\log e |e^x + e^{-x}| + k$

(c) $\log_e |e^x - e^{-x}| + k$

(d) none of these



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CODE : SS12

Choose the most appropriate option (a) (b) (c) or (d).

Que. 35 If $f'(x) = x - 1$, the equation of a curve $y = f(x)$ passing through the point $(1, 0)$ is given by

- (a) $y = x^2 - 2x + 1$
- (b) $y = x^2/2 - x + 1$
- (c) $y = x^2/2 - x + \frac{1}{2}$
- (d) none of these



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MATHS

Application of Integration to Commerce And Economics:

Determination of cost function

- If C denotes the total cost and $MC = \frac{dC}{dx}$ is the marginal cost, then we can write $C = C(x) = \int (MC) dx + k$, where k is the constant of integration, k , being the constant, is the fixed cost.

Example The marginal cost function of manufacturing x units of a product is $5 + 16x - 3x^2$.

The total cost of producing 5 times is Rs. 500. Find the total cost function.

Example The marginal cost (MC) of a product is given to be a constant multiple of number of units (x) produced. Find the total cost function, if Fixed cost is Rs. 5000 and the cost of producing 50 units is Rs. 5625

Determination of Total Revenue Function:

- If $R(x)$ denote the total revenue function and MR is the marginal revenue function, then

$$MR = \frac{d}{dx}[R(x)]$$

$$\therefore R(x) = \int (MR)dx + k \text{ Where } k \text{ is the constant of integration.}$$

Also, when $R(x)$ is known, the demand function can be found as $p = \frac{Rx}{x}$

Example The marginal revenue function for a product is given by

$$MR = \frac{6}{(x-3)^2} - 4.$$

Find the total revenue function and the demand function.

2010 – DECEMBER

The cost function for the production of x units of a commodity is given by

$$C(x) = 2x^3 - 15x^2 + 36x + 15$$

The cost will be minimum when ' x ' is equal to

(a) 3

(b) 2

(c) 1

(d) 4

2017 - JUNE

The equation of the curve which passes through the point (1,2) and has the slope $3x - 4$ at any point (x,y) is:

(a) $2y = 3x^2 - 8x + 9$ (b) $y = 6x^2 - 8x + 9$

(c) $y = x^2 - 8x + 9$ (d) $2y = 3x^2 - 8x + 9$

2019 – JUNE

If the given cost function of commodity is given by $C = 150x - 5x^2 + x^3/6$

, where C stands for cost and x stands for output, if the average cost is equal to the marginal cost then the output x = _____.

(a) 5

(b) 10

(c) 15

(d) 20

The average cost function of a good is $2Q + 6 + 13/Q$ where Q is the quantity produced. The approx cost at 15 is:

- (a) 42**
- (b) 36**
- (c) 66**
- (d) 130**

2021 – JANUARY

The cost function of production is given by $C(x) = x^3/2 - 15x^2 + 36x$ where x ,

Denotes the number of items produced.

The level of output for which marginal cost is minimum and the level of output for which the average cost is minimum are given by, respectively.

(a) 10 and 15

(b) 10 and 12

(c) 12 and 15

(d) 15 and 10

2021 – JULY

In a market there are 30 shops to allocate to people. If they allocate. x shops then their monthly expenses, in rupees, is given by $p(x) = - 8x^2 + 400 x - 1000$, then the number of shops should they allocate to minimize the expenses:

- (a) 0**
- (b) 30**
- (c) 25**
- (d) 10**

2021 - DECEMBER

■ The cost for producing x units is $500 - 20x^2 + x^3/3$. The marginal cost is minimum at $x =$ _____.

- (a) 5
- (b) 10
- (c) 40
- (d) 50